Radar Imaging Homework 1

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Introduction

We are given a system with the scenario depicted in Figure 1. A UAV radar illuminates a scene with three targets placed in different ground range positions.

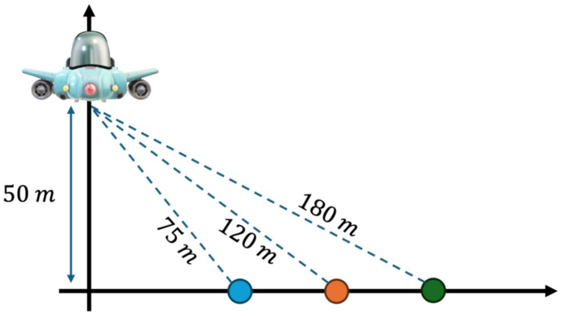


Figure 1: System Geometry

Given Information

It is given that all the targets are placed at boresight for simplicity. So, $f_{tx} = f_{rx} = 1$. All the other system parameters are mentioned in the table present in the material we were provided.

Note: Transmitted power(P_{TX}) mentioned in the table is used only in the solution of question 4.

1 Derivation of the transmitted power

1.1 From Given Information

We are given the information that the signal-to-noise-ratio before range compression is given to be 0 dB. We also have to consider the target at the range of 180 m with a $\sigma = 15\ m^2$.

$$T_{\text{sys}} = T_{\text{scene}} + T_{\text{rec}} = 578.6261\,[\text{K}]$$

$$\Delta\theta_{\text{rad}} = 40^\circ \cdot \frac{\pi}{180} \approx 0.698\,[\text{rad}]$$

$$\Delta\psi_{\text{rad}} = 40^\circ \cdot \frac{\pi}{180} \approx 0.698\,[\text{rad}]$$

1.2 To Derive

We are asked to derive the transmitted power in dBm for a the above mentioned requirements.

1.3 Derivation

Antenna Gain and Effective Area

Approximate solid angle:

$$\Omega_A = \Delta\theta_{\text{rad}} \times \Delta\psi_{\text{rad}} \approx 0.698 \times 0.698 \approx 0.487 \,[\text{sr}] \tag{1}$$

Gain:

$$G \approx \frac{4\pi}{\Omega_A} \approx \frac{4\pi}{0.487} \approx 25.8 \tag{2}$$

Effective Aperture:

$$A_e = \frac{\lambda^2 G}{4\pi} = \frac{(0.03)^2 \cdot 25.8}{4\pi} \approx 1.847 \times 10^{-3} \,[\text{m}^2] \tag{3}$$

Incident Power Density

$$S_i = \frac{P_{\text{Tx}}\, G_{\text{Tx}}\, f_{\text{Tx}}(\theta_0, \psi_0)}{4\pi R_0^2} \tag{4}$$

Solving for transmit power:

$$P_{\text{Tx}} = \frac{S_i\, 4\pi R_0^2}{G_{\text{Tx}} f_{\text{Tx}}(\theta_0, \psi_0)} \tag{5}$$

Scattered Power

$$P_s = S_i \sigma \tag{6}$$

Scattered power density at receiver:

$$S_r = \frac{P_s}{4\pi R_0^2} = \frac{S_i \sigma}{4\pi R_0^2} \tag{7}$$

Received Power

$$P_{\text{Rx}} = S_r\, A_e\, f_{\text{Rx}}(\theta_0, \psi_0) \tag{8}$$

Substitute Eq 7 in Eq 8:

$$P_{\text{Rx}} = \frac{S_i \sigma}{4\pi R_0^2}\, A_e\, f_{\text{Rx}}(\theta_0, \psi_0) \tag{9}$$

Solving Eq 9 for S_i:

$$S_i = \frac{P_{\text{Rx}}\, 4\pi R_0^2}{\sigma A_e\, f_{\text{Rx}}(\theta_0, \psi_0)}$$

Noise and SNR

Given SNR = 0 dB:

$$\text{SNR} = \frac{P_{\text{Rx}}}{P_N} = 1 \Rightarrow SNR = P_N = P_{RX} \tag{10}$$

Noise power:

$$P_N = P_{RX} = kT_{\text{sys}}B$$

Solving for P_N, we get,

$$P_N = 1.38 \times 10^{-23} \times 578.6261 \times 100 \times 10^6 \approx 7.9850 \times 10^{-13} \text{ [W]}$$

$$P_{RX} \approx 7.9850 \times 10^{-13} \text{ [W]} \tag{11}$$

Directivity function is given to be

$$f_{RX}(\theta_o, \phi_o) = f_{TX}(\theta_o, \phi_o)$$

$$f_{TX}(\theta_o) \cdot f_{TX}(\phi_o) = f_{RX}(\theta_o) \cdot f_{RX}(\phi_o) \tag{12}$$

We know the elevation of the radar is 45° and we need to find the elevation of the target at $R_0 = 180[\text{m}]$,

$$cos^{-1}(H/R_0) = cos^{-1}(50/180) \approx 1.28[\text{rad}] \approx 73.87^\circ$$

Adjusting elevation after with the elevation of the radar we get,

$$\theta = 28.8^\circ \approx 0.50[\text{rad}] \tag{13}$$

We know the target is in boresight so we can assume $\phi = 0^\circ$.

Substituting for result obtained in Eq.13 in Eq.12 we get,

$$f_{TX}(\theta_o, \phi_o) = \operatorname{sinc}(\theta)^2 \times \operatorname{sinc}(\phi)^2$$

$$f_{TX}(\theta_o, \phi_o) = \operatorname{sinc}(28.8^\circ)^2 \times 1^2 \approx 0.633^2 \times 1 = 0.4053 \tag{14}$$

$$S_i = \frac{7.9850 \times 10^{-13} \times 4\pi \times 180^2}{15 \times 1.847 \times 10^{-3} \times 0.4053} \tag{15}$$
$$= 2.8953 \times 10^{-5} \text{ [W]}$$

$$P_{TX} = \frac{2.8953 \times 10^{-5} \times 4\pi \times 180^2}{25.8 \times 0.4053} \tag{16}$$
$$= 1.1273 \text{ [W]}$$

$$= 10 \log_{10}(1.1273) = 0.5205 \text{ dBW} + 30 \tag{17}$$
$$= 30.5205 \text{ [dBm]}$$

2 Matlab Implementation:

2.1 Given Information:

We are provided with the P_{tx} and all the other values needed to find the Signal-to-Noise Ratio(SNR).

2.2 To Implement:

We need to implement the equations from the provided material to calculate the SNR before range compression.

2.3 Implementation:

The implementation can be found in the attached file (Question2.m).

2.4 Results:

A screenshot of the obtain results after the execution of the Matlab file has been provided in the Figure 2

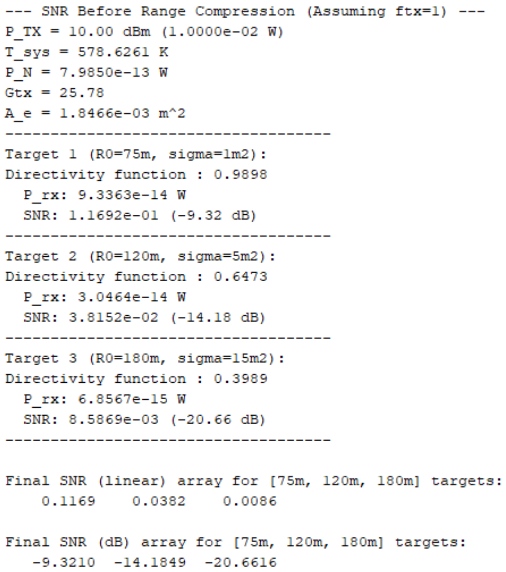
3 Derivation of SNR After Range Compression

Given Information

We are provided with all the radar parameters.

To solve:

We are asked to derive the SNR after range compression.

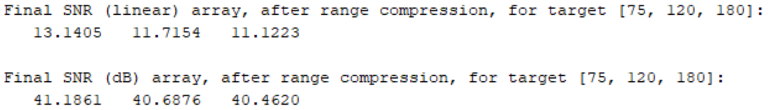


```
--- SNR Before Range Compression (Assuming ftx=1) ---
P_TX = 10.00 dBm (1.0000e-02 W)
T_sys = 578.6261 K
P_N = 7.9850e-13 W
Gtx = 25.78
A_e = 1.8466e-03 m^2
------------------------------------
Target 1 (R0=75m, sigma=1m2):
Directivity function : 0.9898
  P_rx: 9.3363e-14 W
  SNR: 1.1692e-01 (-9.32 dB)
------------------------------------
Target 2 (R0=120m, sigma=5m2):
Directivity function : 0.6473
  P_rx: 3.0464e-14 W
  SNR: 3.8152e-02 (-14.18 dB)
------------------------------------
Target 3 (R0=180m, sigma=15m2):
Directivity function : 0.3989
  P_rx: 6.8567e-15 W
  SNR: 8.5869e-03 (-20.66 dB)
------------------------------------

Final SNR (linear) array for [75m, 120m, 180m] targets:
    0.1169    0.0382    0.0086

Final SNR (dB) array for [75m, 120m, 180m] targets:
   -9.3210  -14.1849  -20.6616
```

Figure 2: SNR obtained from the theoretical calculations. [Question 2]



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Final SNR (linear) array, after range compression, for target [75, 120, 180]:
   13.1405   11.7154   11.1223

Final SNR (dB) array, after range compression, for target [75, 120, 180]:
   41.1861   40.6876   40.4620
```

Figure 3: SNR obtained from the Matlab simulation [Question 4]

Derivation

The received power is derived by substitution in Equation 5:

$$P_{RX} = P_{TX} \frac{\sigma \lambda^2 G^2 f_{TX}^2}{(4\pi)^3 R^4} \tag{18}$$

The signal energy at the matched filter input is $E_s = P_{RX} T_p$, where T_p is the pulse duration. The matched filter output peak is $|y_{sig}(t_0)| = E_s$. The signal power at the output peak (amplitude squared) is:

$$P_{signal,after} = |y_{signal}(t_0)|^2 = (P_{RX} T_p)^2 \tag{19}$$

The power spectral density of the thermal noise is obtained by,

$$S_n(f) = N_0 = kT_{sys}$$

For a matched Filter the power of the output noise is obtained by using Fourier Transform,

$$P_{noise,after} = kT_{sys} \int |h(t)|^2 dt = kT_{sys} P_{RX} T_p \tag{20}$$

The output SNR is the ratio of $P_{signal,after}$ to $P_{noise,after}$:

$$SNR_{after} = \frac{P_{signal,after}}{P_{noise,after}} = \frac{(P_{RX} T_p)^2}{kT_{sys} P_{RX} T_p} = \frac{P_{RX} T_p}{kT_{sys}} \tag{21}$$

Substituting P_{RX} from the first step:

$$SNR_{after}(R, \sigma) = \frac{P_{TX} \sigma \lambda^2 G^2 f_{TX}^2 T_p}{(4\pi)^3 R^4 k T_{sys}} \tag{22}$$

The SNR before compression is $SNR_{before} = \frac{P_{RX}}{kT_{sys}B}$, where B is the bandwidth.
Taking the ratio of SNR_{after} to SNR_{before}:

$$\frac{SNR_{after}}{SNR_{before}} = \frac{P_{RX} T_p / (kT_{sys})}{P_{RX} / (kT_{sys} B)} = BT_p \tag{23}$$

Thus, the final relation shows the SNR gain from matched filtering:

$$SNR_{after} = SNR_{before}\ BT_p \tag{24}$$

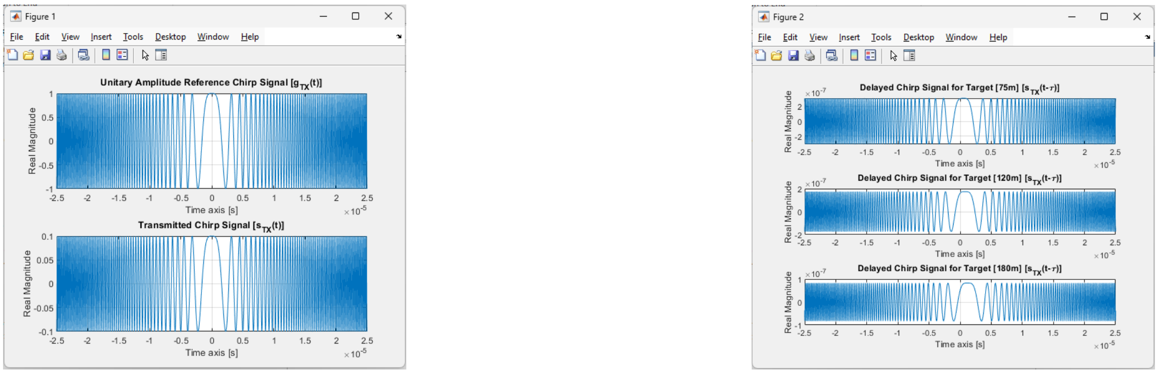
4 Matlab Simulation:

4.1 Given:

The steps to be performed in the simulation is given to us.

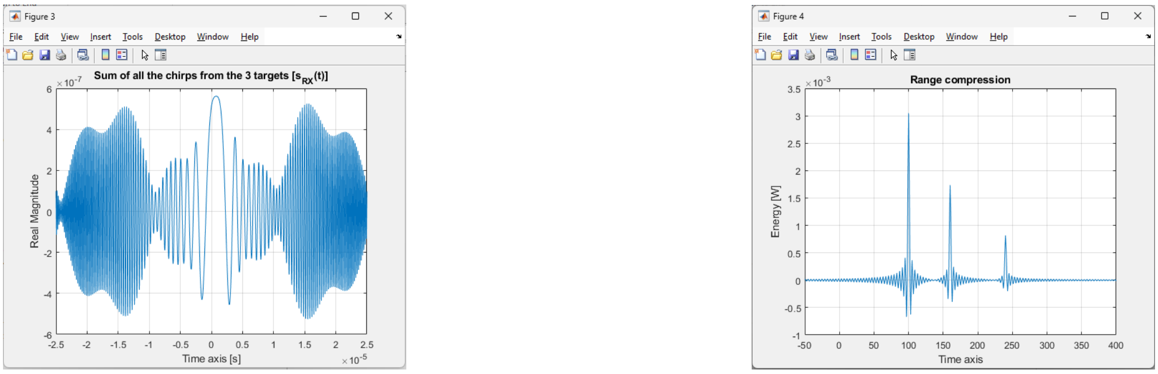
4.2 Results of the simulation:

The results of the simulation is shown in Figure 3 and Figure 4.



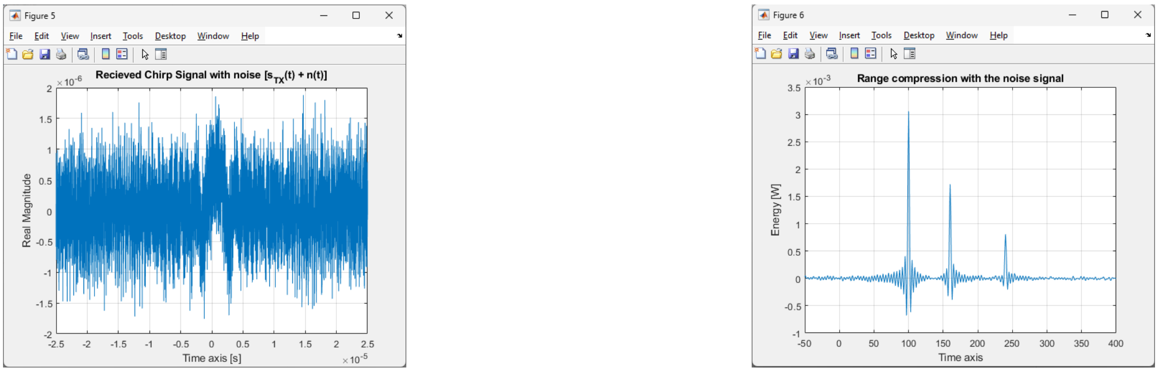
(a) Unit amplitude chirp and transmitted chirp signal

(b) Delayed, demodulated, attenuated signals



(c) Sum of the 3 reflected signals

(d) Range compression



(e) Sum of the 3 reflected signals with noise

(f) Range compression of the signals with noise

Figure 4: Results of the simulation

Note: The Matlab Implementations are attached with the report.